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SHORTER COMMUNICATION

ANALYSIS OF HEAT-TRANSFER FROM A VERTICAL PLANE SURFACE BY TURBULENT NATURAL CONVECTION

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NOMENCLATURE

St,	Stanton number;
C_f,	drag coefficient;
G,	Grashof number;
σ,	Prandtl number;
N,	local Nusselt number;
$\bar{N}$,	Nusselt number for mean heat-transfer;
ψ,	defined within;
t,	defined within;
t_0,	defined within;
K,	$= 2.5$, constants of the universal
η_0,	$= 11.6$, velocity profile;
ln,	Napierian logarithm.

INTRODUCTION

EXPRESSIONS for the drag on plates, or for pipe friction, due to turbulent flow gained a dramatic increase in range of validity when power-law expressions (depending on the seventh root velocity distribution) were replaced by the Schoenherr–von Kármán logarithmic laws (based on the universal logarithmic velocity distribution). Moreover, a three-part piecewise analytic version of the universal velocity distribution leads to

$$\frac{1}{St} = \frac{2}{C_f} + \sqrt{\left(\frac{2}{C_f}\right)}\psi(\sigma)$$

wherein, according to von Kármán,

$$\psi(\sigma) = 5\{(\sigma-1)+\ln[1+5(\sigma-1)/6]\},$$

although other valid expressions for ψ exist. All this is clear from standard works [1, 2].

Eckert and Jackson [3] gave, for turbulent natural convection, an analysis based on similar profiles tending to the seventh root law near the wall. We here briefly report a successful application of the universal velocity profile to this problem.

PRINCIPLES OF THIS ANALYSIS

It is assumed that the velocity distribution in the boundary layer follows the established universal law up to the point at maximum velocity. The related temperature distribution is used throughout, continuing linearly on the temperature-ln (distance) plane up to the outer edge of the boundary layer. It is postulated that the velocity distribution is linear on the velocity-ln (distance) plane beyond the maximum, but with a reversed gradient compared with that before the maximum.

These profiles, substituted in the equations for the momentum and energy integrals for the boundary layer, lead to inhomogeneous non-linear ordinary differential equations for which asymptotic solutions may be found.

RESULT OF ANALYSIS

It is found that

$$N = \frac{1}{K^2}\left[\frac{G\sigma^2}{7t(t_0)^3}\right]^{1/2}$$

where

$$t_0 = \tfrac{1}{2}\ln\left(\frac{G}{63K^6\eta_0^2}\right) + \frac{1}{K}[\eta_0+\psi(\sigma)].$$

K and η_0 are constants of the universal velocity profile and the function $t(t_0)$ must satisfy

$$t_0 = t + \ln(t)/2.$$

It follows that

$$\bar{N}/N \simeq (2t+1)/(3t-3).$$

On inserting the constants and using an approximate solution for $t(t_0)$, we have the practical result

$$N = 0.0605[G\sigma^2/t^3]^{1/2},$$
$$t = t_0 - \tfrac{1}{2}\ln(0.9t_0),$$
$$t_0 = \tfrac{1}{2}\ln\left(\frac{G}{193}\right) + \tfrac{2}{5}\psi(\sigma).$$

COMPARISON WITH EXPERIMENT

Values of N calculated from the theory agree well with Cheesewright's [4] observations, using air, for $10^{10} < G < 10^{11}$. For $G = 10^{12}$ and $\sigma = 1$ we find that $t_0 = 11.2$ and $t = 10.0$. Thence, we predict $N = 1920$ and $\bar{N} = 1500$. This agrees with Jakob's correlation [5].

DISCUSSION

That N should tend to a function of the Boussinesq number $G\sigma^2$, for large G or small σ has long been known. Indeed the ultimate cancellation of viscosity and thermal conductivity requires that N should become asymptotically proportional to $\sigma\sqrt{G}$, as found here. That N fails to become a function of the Rayleigh number, $G\sigma$, for large σ may be, at first, surprising. The theory is found to suggest that the critical Grashof number for the onset of turbulence is roughly $6\times10^9\{1+\psi(\sigma)/19\}$. Crude though this estimate be, it is clearly dominated by $\psi(\sigma)$—and hence by σ—for large σ. So, if σ tends to infinity, then turbulence is unattainable for finite G.

It is well known that the use of the boundary-layer integrals can lead to good results for heat-transfer even when the profile is erroneous. We therefore do not claim that the successful results confirm the assumed profile.

On the other hand, since the mathematical approximations made become ever more valid the higher the value of G, and since, as has already been seen, the asymptotic behaviour for large G accords with long-recognized general principles, we make bold to claim that our result is well suited for indefinite upward extrapolation. We know of no other result free from disastrous failure as G is increased without bound.

We hope to report this work *in extenso* during 1976.

I dedicate this brief communication to the memory of my old friend and colleague, Allan Ede.

REFERENCES

1.* S. Goldstein (editor), *Modern Developments in Fluid Dynamics*. Clarendon Press, Oxford (1938).
2.* L. Howarth, *Modern Developments in Fluid Dynamics: High Speed Flow*. Clarendon Press, Oxford (1953).

*See, in particular, the contributions by H. R. Squire in [1] and [2].

3. E. R. G. Eckert and T. W. Jackson, Analysis of turbulent free-convection boundary layer on flat plate, Nat. Advisory Comm. Aeronaut. Tech. Note 2207 (October 1950).
4. R. Cheesewright, Turbulent natural convection from a vertical plane surface, *J. Heat Transfer* **90**, 1–8 (1968).
5. M. Jakob, *Heat Transfer*, Vol. 1, p. 529. John Wiley, New York (1949).